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$\mathrm{SU}_L(4) \times \mathrm{U}(1)$ model for electroweak unification

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Abstract: After some general remarks about $\mathrm{SU}_L(4)$ electroweak unification, the model is extended to $\mathrm{SU}_L(4) \times \mathrm{U}_X(1)$ to accomodate fractionally charged quarks. The unification scale is expected to be in TeV region. A right-handed Majorana neutrino along with known lepton are put in the fundamental representation of $\mathrm{SU}_L(4)$ with $Y_X = 0$. The see-saw mechanism for neutrino masses and flavor mixing in neutrino sector is a natural feature of the model. The lepton number violating processes can occure through dilepton gauge bosons contained in the model.

Keywords: Gauge Symmetry, Beyond Standard Model, Neutrino Physics.

Contents

1. Introduction

It is well-known that the standard $\mathrm{SU}_L(2) \times \mathrm{U}(1)$ model does not provide a true unification of electroweak interaction, since because of the U(1) factor, $\sin^2\theta_W$ is not fixed by the model. The extensions of the standard model to higher groups such as SU(5), $O(10)$ or Pati-Salam group $\mathrm{SU}_c(4) \times \mathrm{SU}_L \times S_R(2)$ do give quantisation of charge and predict $\sin^2\theta_W = 3/8$ at unification scale; this unification scale is of order 10^{16} GeV [1, 2, 3]. The reason for the huge desert is the large disparity between the measured value of $\sin^2\theta_W = 0.231$ and $\sin^2\theta_W = 3/8$ at the unification scale. The most attractive model in this catagory of unification is the supersymmetric standard model based on the group SU(5) (SSM) [4].

There are other models based on the simple groups SU(3) or SU(4), which gives $\sin^2\theta_W = 1/4$ at the unification scale. For the group $\mathrm{SU}_L(3)$ [5, 6, 7], the quantised charge Q can be written as

$$Q = \frac{1}{2}\left[\lambda_3 + \sqrt{3}\lambda_8\right] = T_3 + \sqrt{3}T_8 = T_3 + \frac{1}{2}Y\,. \tag{1.1}$$

Thus leptons can be assigned naturally to the representation

$$3_R = \begin{pmatrix} e^+ \\ \nu_e^c \\ e^- \end{pmatrix}_R \Longrightarrow 3_L^* = \begin{pmatrix} e^- \\ \nu_e \\ e^+ \end{pmatrix}_L \tag{1.2}$$

of $\mathrm{SU}_L(3)$. For this case

$$\begin{aligned} \frac{A_\mu}{e} &= \frac{W_{3L\mu}}{g} + \frac{B_\mu}{g}, \qquad (1.3)\\ \frac{1}{e^2} &= \frac{1}{g^2} + \frac{1}{g'^2} \\ \tan\theta_W &= \frac{g'}{g}\,. \qquad (1.4)\end{aligned}$$

JHEP12(2004)013

Since in the symmetry limit $g' = g/\sqrt{3}$, it gives at the unification scale $\sin^2\theta_W = 1/4 = 0.25$ near to the measured value and thus unification scale is expected to be in TeV region. This is an attractive feature of this group. However, quarks cannot be accomodated in this model. It needs to be extended to $\mathrm{SU}_L(3) \times \mathrm{U}_X(1)$ in order to accomodate the quarks. The anomely free model based on the group $\mathrm{SU}_L(3) \times \mathrm{U}_X(1)$ has been considered by several authors [6, 7, 8].

A unification model based on the group $\mathrm{SU}_L(4)$ has been considered in ref. [9]. It not only accomodates leptons nicely; but also a right-handed Majorana neutrino. The see-saw mechanism for generation of masses of the standard model neutrinos is a natural feature of this model. The charge operator in this model is given by

$$Q = \frac{1}{2}\left[\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right] + \sqrt{\frac{2}{3}}\lambda_{15}\,. \tag{1.5}$$

The leptons can be assigned to the fundamental representation of this group (Ist generation) as follows:

$$F_e = \begin{pmatrix} e^+ \\ \nu_e^c \\ -N_e \\ e^- \end{pmatrix}_R : 4_R\,. \tag{1.6}$$

It is convienent to write the charge operator in the form

$$Q = \frac{1}{2}\left[\tau_{3L} + \tau_{3R} + Y_1\right] \tag{1.7}$$

where

$$\begin{aligned} \lambda_3 &= \begin{pmatrix} \tau_{3L} & 0 \\ 0 & 0 \end{pmatrix}, \\ \frac{1}{\sqrt{3}}\left(-\lambda_8 + \sqrt{2}\lambda_{15}\right) &= \begin{pmatrix} 0 & 0 \\ 0 & \tau_{3R} \end{pmatrix} \end{aligned} \tag{1.8}$$

and

$$Y_1 = \frac{1}{\sqrt{3}}\left(2\lambda_8 + \sqrt{2}\lambda_{15}\right) = \mathrm{diag}\,(1,1,-1,-1)\,. \tag{1.9}$$

The vector bosons belong to the adjoint representation of $\mathrm{SU}_L(4)$: (suppressing the Lorentz index μ)

$$W = \frac{1}{\sqrt{2}}\begin{pmatrix} W_{3L} + \frac{1}{\sqrt{2}}B_1 & \sqrt{2}W_L^- & \sqrt{2}X_1^- & \sqrt{2}Y_1^{--} \\ \sqrt{2}W_L^+ & -W_{3L} + \frac{1}{\sqrt{2}}B_1 & \sqrt{2}X_2^0 & \sqrt{2}Y_2^- \\ \sqrt{2}X_1^+ & \sqrt{2}\bar{X}_2^0 & W_{3R} - \frac{1}{\sqrt{2}}B_1 & \sqrt{2}W_R^- \\ \sqrt{2}Y_1^{++} & \sqrt{2}Y_2^+ & \sqrt{2}W_R^+ & -W_{3R} - \frac{1}{\sqrt{2}}B_1 \end{pmatrix}.$$

Thus

$$\frac{A_\mu}{e} = \frac{W_{3L\mu}}{g} + \frac{W_{3R\mu}}{g} + \frac{B_{1\mu}}{g_1} = \frac{W_{3L\mu}}{g} + \frac{B_\mu}{g'} \tag{1.10}$$

$$\frac{B_\mu}{g'} = \frac{W_{3R\mu}}{g} + \frac{B_{1\mu}}{g_1} \tag{1.11}$$

$$\frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2} \tag{1.12}$$

$$\frac{1}{g'^2} = \frac{1}{g^2} + \frac{1}{g_1^2}\,. \tag{1.13}$$

In the symmetry limit $g_1 = g/\sqrt{2}$ and hence at the unification scale $g_1 = g/\sqrt{3}$, and $\sin^2\theta_W = 1/4$.

Before we proceed further, it is interesting to see that in $\mathrm{SU}_L(4)$, there is an alternative way to write the charge operator:

$$\begin{aligned} Q &= \frac{1}{2}\left[\lambda_3 - \frac{1}{3\sqrt{3}}\lambda_8 + \frac{4}{3}\sqrt{\frac{2}{3}}\lambda_{15}\right] \\ &= \frac{1}{2}\left[\tau_{3L} + \tau_{3R} + Y_1\right] \end{aligned} \tag{1.14}$$

where

$$Y_1 = \frac{1}{3\sqrt{3}}\left(2\lambda_8 + \sqrt{2}\lambda_{15}\right). \tag{1.15}$$

In this case $Y_1 = \mathrm{diag}\,(1/3, 1/3, -1/3, -1/3)$. It is then natural to assign the quarks to the fundamental representation of $\mathrm{SU}_L(4)$ as follows

$$F_q = \begin{pmatrix} u \\ d \\ d^c \\ u^c \end{pmatrix}_L : 4_L\,. \tag{1.16}$$

However, for this case, at the unification scale $\sin^2\theta_W = 9/20 = 0.45$; hence the unification scale may be as high as Planck mass. $\Delta B = 2$ transition will be highly suppressed. We conclude that for $\mathrm{SU}_L(4)$, the natural choice for charge operator is that given in eq. (1.4); the leptons belong to the fundamental representation of $\mathrm{SU}_L(4)$, the unification scale is in TeV region; the lepton number violation occurs at TeV scale; Majorana mass for the right-handed neutrino is also large and see-saw mechanism is a natural consequence of $\mathrm{SU}_L(4)$.

We note that representation (1.6) is not an anomaly free. An anomaly free model can be constructed by putting Hahn and Nambu quarks [10] with integral gauge charges in the fundamental representation of $\mathrm{SU}_L(4)$ as follows (for first generation)

$$\begin{aligned} F_d^r &= \begin{pmatrix} d'^c \\ u^c \\ -u \\ d' \end{pmatrix}_R^r : 4_R \Longrightarrow 4_L^* \\ F_d^{g,b} &= \begin{pmatrix} u \\ -d \\ d^c \\ u^c \end{pmatrix}_L^{g,b} : 4_L\,. \end{aligned} \tag{1.17}$$

It is clear that representations (1.6) and (1.17) together cancel the anomalies. The gauge symmetry $\mathrm{SU}_L(4)$ is broken as follows

$$\mathrm{SU}_L(4) \underset{m_{X,Y}}{\rightarrow} \mathrm{SU}_L(2) \times \mathrm{SU}_R(2) \times \mathrm{U}_1(1) \underset{m_R}{\rightarrow} \mathrm{SU}_L(2) \times \mathrm{U}_1(1) \underset{m_L}{\rightarrow} \mathrm{U}_{em}(1)\,. \tag{1.18}$$

As we have emphasized earlier this model works well for leptons. For quarks, the following comments are in order. The electromagnetic current can be split into two parts viz, the color singlet (containing the fractionally charged quarks) and a color octet. The currents coupled to X and Y bosons do not contain any color singlets. Thus with the singlet part of the electromagnet current; it effectively reduces to the standard model of electroweak interactions [9], since known hadrons are color singlets. However, this model becomes more appealing if we could formulate this model in higher dimensions and gauge symmetry is broken by orbifold compactification to four dimensions so that X and Y bosons and non-color singlet part of electromagnetic current is relagated to the bulk and we have effectively fractional quarks on the brane.

2. Symmetry group $\mathrm{SU}_L(4) \times \mathrm{U}_X(1)$

In order to accomodate the quarks with fractional charges, it is necessary to extend the group to $\mathrm{SU}_L(4) \times \mathrm{U}_X(1)$.

However, for the extended group $\mathrm{SU}_L(4) \times \mathrm{U}_X(1)$, true unification is lost and $\sin^2\theta_W$ is arbitrary. Neverthless for this group, $\sin^2\theta_W$ comes to be less than 1/4 and it is still possible to have unification mass scale in TeV region. To see this we note that for the extended group we have an extra gauge boson $V_\mu(x)$ with gauge coupling constant g_X. In the limit $g_X \to \infty$, the vector boson $V_\mu(x)$ is decoupled, we have effectively the group $\mathrm{SU}_L(4)$, with the unification mass scale $m_X \sim 5.8\,\mathrm{TeV}$ and $\sin^2\theta_W(m_X) = 1/4$. However with finite g_X, we have $\sin^2\theta_W(m_X) < 1/4$, $m_X < 5.8\,\mathrm{TeV}$. The lower bound on unification mass scale would be determined by the experimental limit on lepton number violating processes. However, the smallness of neutrino masses, would require the Majorana mass of right-handed neutrino to be in TeV region. Thus one would expect the unification mass scale m_X to be few TeV. In fact we find for $\alpha_2(m_Z)/\alpha_X(m_Z) \approx 0.147$, $m_X = 960\,\mathrm{GeV}$, where as for $\alpha_2(m_Z)/\alpha_X(m_Z) \approx 0.052$, $m_X = 3\,\mathrm{TeV}$.

As long as $g_X(m_Z)/g_2(m_Z) \gg 1$, it is still possible to have a sort of unification at a TeV mass scale (For example, for $g_X(m_Z)/g_2(m_Z) \approx 4.4$, $m_X = 3\,\mathrm{TeV}$).

We now give the details of the model. For anomely cancelation, the leptons and quarks must be assigned as follows:

$$4_R: \begin{pmatrix} e^+ \\ -\nu_e^c \\ N_e' \\ e^- \end{pmatrix}_R, \begin{pmatrix} \mu^+ \\ -\nu_\mu^c \\ N_\mu' \\ \mu^- \end{pmatrix}_R, \begin{pmatrix} \tau^+ \\ -\nu_\tau^c \\ N_\tau' \\ \tau^- \end{pmatrix}_R \Rightarrow 4_L^* \qquad Y_X = 0$$

$$4_L: \begin{pmatrix} u^a \\ d'^a \\ D^a \\ H_d^a \end{pmatrix}_L, \begin{pmatrix} c^a \\ s'^a \\ S^a \\ H_s^a \end{pmatrix}_L, \qquad Y_X = -\frac{2}{3}$$

JHEP12(2004)013

$$
\begin{array}{lllllllll}
I_R: (u^a)_R\,, & (d'^a)_R\,, & (D^a)_R\,, & (H_d^a)_R\,, & (c^a)_R\,, & (s'^a)_R\,, & (S^a)_R\,, & (H_s^a)_R \\
Y_X = \;\; 4/3 & -2/3 & -2/3 & -8/3 & 4/3 & -2/3 & -2/3 & -8/3
\end{array} \tag{2.1}
$$

$$
4_R: \begin{pmatrix} b'^a \\ t^a \\ U^a \\ T^a \end{pmatrix}_R^c , \qquad Y_X = -\frac{4}{3}
$$

$$
\begin{array}{llll}
I_L: (b'^a)_L^c\,, & (t'^a)_L^c\,, & (U^a)_L^c\,, & (T_d^a)_L^c \\
Y_X = \;\; +2/3 & -4/3 & -4/3 & -10/3
\end{array} \tag{2.2}
$$

Here $a = 1, 2, 3$ is the color index. Thus we have six extra quarks (exotic) with charges

$$
\begin{aligned}
U, D, S: &\quad (2/3, -1/3, -1/3) \\
H_d, H_s, T: &\quad (-4/3, -4/3, 5/3)\,.
\end{aligned} \tag{2.3}
$$

The photon A_μ and boson B_μ associated with $\mathrm{U}_Y(1)$ are given by

$$
\begin{aligned}
\frac{A_\mu}{e} &= \frac{W_{3L\mu}}{g} + \frac{W_{3R\mu}}{g} + \frac{B_{1\mu}}{g_1} + \frac{V_\mu}{g_X} = \frac{W_{3L\mu}}{g} + \frac{B_\mu}{g'} \\
\frac{B_\mu}{g'} &= \frac{W_{3R\mu}}{g} + \frac{B_{1\mu}}{g_1} + \frac{V_\mu}{g_X}
\end{aligned} \tag{2.4}
$$

$$
\begin{aligned}
\frac{1}{e^2} &= \frac{2}{g^2} + \frac{1}{g_1^2} + \frac{1}{g_X^2} = \frac{2}{g^2} + \frac{1}{g'^2} \\
\frac{1}{g'^2} &= \frac{1}{g^2} + \frac{1}{g_1^2} + \frac{1}{g_X^2}
\end{aligned} \tag{2.5}
$$

In the symmetry limit of $\mathrm{SU}_L(4)$, $g_1 = g/\sqrt{2}$; thus we get

$$
\frac{1}{g'^2} = \frac{3}{g^2} + \frac{1}{g_X^2} \tag{2.6}
$$

$$
\begin{aligned}
\frac{g^2}{g_X^2} &= \frac{1 - 3\tan^2\theta_W\,(m_X)}{\tan^2\theta_W\,(m_X)} \\
\frac{g_X^2}{g^2} &= \frac{\sin^2\theta_W\,(m_X)}{1 - 4\sin^2\theta_W\,(m_X)}\,.
\end{aligned} \tag{2.7}
$$

Thus $\sin^2\theta_W\,(m_X) < 1/4$, implying the unification scale to be in TeV range. For $\sin^2\theta_W\,(m_X) = 1/4$, $g_X \to \infty$ and V_μ will decouple at m_X.

Here it is relevant to give an estimate of the unification scale. We first consider the unification group $\mathrm{SU}_L(4)$ and its breaking direct to the group $\mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$. A straight forward application of renormalization group equations give [9]

$$
\alpha^{-1}(m_Z)\left[1 - 4\sin^2\theta_W\right] = 2\left(-C_1^2\beta_2 + \beta_1\right)\ln\frac{m_X}{m_Z} \tag{2.8}
$$

where

$$
\begin{aligned}
\beta_2 &= \frac{1}{4\pi}\left[-\frac{22}{3} + \frac{4}{3}\frac{n_f}{2}\right] \\
\beta_1 &= \frac{1}{4\pi}\left[\frac{4}{3}\frac{n_f}{2}C_1^2\right] \\
C_1^2 &= 2\,.
\end{aligned} \tag{2.9}
$$

JHEP12(2004)013

Using $\sin^2\theta_W(m_Z) = 0.2311$, $\alpha^{-1}(m_Z) = 128$, we get unification mass scale

$$m_X = 63.1 m_Z \simeq 5.8\,\mathrm{TeV}\,. \tag{2.10}$$

We now consider the group $\mathrm{SU}_L(4) \times \mathrm{U}_X(1)$. For this case, the renormalization group equations give

$$\begin{aligned}
\alpha_L^{-1}(m_Z) &= \alpha_G^{-1} + 2\beta_{2L} \ln \frac{m_X}{m_Z} \\
\alpha_R^{-1}(m_Z) &= \alpha_G^{-1} + 2\beta_{2R} \ln \frac{m_X}{m_Z} \\
\alpha_1^{-1}(m_Z) &= C_1^2 \alpha_G^{-1} + 2\beta_1 \ln \frac{m_X}{m_Z} \\
\alpha_R^{-1}(m_Z) + \alpha_1^{-1}(m_Z) &= \left(1 + C_1^2\right) \alpha_G^{-1} + 2\left(\beta_{2R} + \beta_1\right) \ln \frac{m_X}{m_Z}\,.
\end{aligned} \tag{2.11}$$

Noting that $\beta_{2L} = \beta_{2R} = \beta_2$, $\alpha_L^{-1} = \alpha_2^{-1}$, and $\alpha_2^{-1}(m_Z) = \alpha^{-1}(m_Z)\sin^2\theta_W$, we obtain

$$\alpha_R^{-1}(m_Z) + \alpha_1^{-1}(m_Z) = \left(1 + C_1^2\right)\alpha^{-1}(m_Z)\sin^2\theta_W(m_Z) + 2\left(-C_1^2\beta_2 + \beta_1\right)\ln\frac{m_X}{m_Z}\,. \tag{2.12}$$

Noting that

$$\alpha'^{\,-1} = \alpha_R^{-1} + \alpha_1^{-1} + \alpha_X^{-1}\,, \tag{2.13}$$

we obtain our final result

$$\frac{\alpha_X^{-1}(m_Z)}{\alpha_2^{-1}(m_Z)} = \frac{1 - 4\sin^2\theta_W}{\sin^2\theta_W} - 2\alpha_2(m_Z)\left(-C_1^2\beta_2 + \beta_1\right)\ln\frac{m_X}{m_Z}\,. \tag{2.14}$$

This relation reduces to eq. (2.8) for $\alpha_X^{-1} = 0$. Thus from eq. (2.10), we conclude that upper bound for the unification mass scale is 5.8 TeV, for which $\alpha_X^{-1} = 0$. The unification scale of order 900 GeV, would correspond to $\alpha_X^{-1}(m_Z)/\alpha_2^{-1}(m_Z) \simeq 0.126$ whereas for the unification mass scale of 3 TeV, this ratio is 0.052.

We note that there are four neutral vector bosons: $W_{3L\mu}$, $B_{1\mu}$, $W_{3R\mu}$, and V_μ. They can be expressed in terms of the photon A_μ, the neutral vector boson Z_μ and two extra neutral bosons Z'_μ and Z''_μ as follows

$$\begin{aligned}
A_\mu &= \sin\theta_W W_{3L\mu} + \cos\theta_W B_\mu \\
Z_\mu &= \cos\theta_W W_{3L\mu} - \sin\theta_W B_\mu \\
Z_{1\mu} &= \frac{g}{g_1} W_{3R\mu} - \frac{g_X}{g_1} V_\mu \\
Z_{2\mu} &= B_{1\mu} - \frac{g_X}{g_1} V_\mu \\
Z'_\mu &= Z_{1\mu} - Z_{2\mu} = \frac{g}{g_1} W_{3R\mu} - B_{1\mu} \\
Z''_\mu &= Z_{1\mu} + Z_{2\mu} = \frac{g}{g_1} W_{3R\mu} + B_{1\mu} - 2\frac{g_X}{g_1} V_\mu\,.
\end{aligned} \tag{2.15}$$

The first stage of symmetry breaking is accomplished by a 15-plet of Higgs scalar Φ:

$$\langle\Phi\rangle = \frac{V}{2}\,\mathrm{diag}\,(1, 1, -1, -1)\,. \tag{2.16}$$

The second and third stages of symmetry breaking are accomplished by introducing 4 Higgs scalars ϕ_1, ϕ_2, ϕ_3, and ϕ_4 belonging to fundamental representation of $\mathrm{SU}_L(4)$ having $Y_X = -2, 0, 0$ and 2 respectively, with the following expectation values

$$\begin{aligned} \langle\phi_1\rangle &= v_1 \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \\ \langle\phi_2\rangle &= v_2 \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \\ \langle\phi_3\rangle &= v_3 \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \\ \langle\phi_4\rangle &= v_4 \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \end{aligned} \tag{2.17}$$

With the symmetry breaking pattern discussed above the mass lagrangian for vector bosons is given by

$$\begin{aligned} \mathcal{L}_{\text{mass}} = {} & \frac{1}{4} g_2^2 V^2 \left[2\bar{X}_1 X_1 + 2\bar{X}_2 X_2 + 2\bar{Y}_1 Y_1 + 2\bar{Y}_2 Y_2\right] + \\ & + \frac{1}{4} g_2^2 v_R^2 \Bigg[2\left(1 + \frac{\mathcal{K}^2 + \mathcal{K}'^2}{v_R^2}\right) W_R^+ W_R^- + 2\sin^2\beta \left(\bar{Y}_1 Y_1 + \bar{Y}_2 Y_2\right) + \\ & \qquad + 2\left(\cos^2\beta + \frac{\mathcal{K}_R^2}{v_R^2}\right)\left(\bar{X}_1 X_1 + \bar{X}_2 X_2\right) + \\ & \qquad + \frac{g_1^2}{g_2^2}\left(\left(\cos^2\beta + \frac{\mathcal{K}_R^2}{v_R^2}\right) Z'^2 + \sin^2\beta Z''^2\right)\Bigg] \\ & + \frac{1}{4} g_2^2 v_L^2 \Bigg\{ 2\left(1 + \frac{\mathcal{K}^2 + \mathcal{K}'^2}{v_L^2}\right) W_L^+ W_L^- + 2\frac{\mathcal{K}^2 + \mathcal{K}'^2}{v_R^2} W_R^+ W_R^- + \\ & \qquad + 2\left(\sin^2\alpha + \frac{\mathcal{K}^2 + \mathcal{K}'^2}{v_L^2}\right) \bar{X}_1 X_1 + 2\left(\cos^2\alpha + \frac{\mathcal{K}^2 + \mathcal{K}'^2}{v_L^2}\right) \bar{Y}_2 Y_2 + \\ & \qquad + 2\left(\sin^2\alpha + \frac{4\mathcal{K}^2}{v_L^2}\right) \bar{Y}_1 Y_1 + 2\left(\cos^2\alpha + \frac{4\mathcal{K}^2}{v_L^2}\right) \bar{X}_2 X_2 + \\ & \qquad + \left(\sin^2\alpha + \frac{\mathcal{K}^2 + \mathcal{K}'^2}{v_L^2}\right) \Bigg[\frac{Z}{\cos\theta_W} + \frac{g_1}{g_2}\left(\tan^2\theta_W - 1\right) + \\ & \qquad\qquad + \frac{1}{2}\frac{g_1}{g_2}\tan^2\theta_W \frac{g^2}{g_X^2}\left(Z' - Z''\right)\Bigg]^2 + \end{aligned}$$

$$+\cos^2\alpha\left[\frac{Z}{\cos\theta_W}+\frac{g_1}{g_2}\tan^2\theta_W\left(Z'+\frac{1}{2}\frac{g_2^2}{g_X^2}\left(Z'-Z''\right)\right)\right]^2+$$
$$\left.+\frac{2\mathcal{K}\mathcal{K}'}{v_L^2}\left[2W_L^+W_R^-+2W_L^-W_R^++2\bar{X}_1Y_2+2X_1\bar{Y}_2\right]\right\} \tag{2.18}$$

where we have put $v_3 = v_R\cos\beta$, $v_4 = v_R\sin\beta$ and $v_1 = v_L\sin\alpha$, $v_2 = v_L\cos\alpha$. The following remarks are in order

1. For the symmetry breaking pattern which we have envsiged viz
$$\begin{aligned}\mathrm{SU}_L(4)\times\mathrm{U}_X(1) &\underset{V^2}{\rightarrow}\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\times\mathrm{U}_1(1)\times\mathrm{U}_X(1)\\ &\underset{v_R^2}{\rightarrow}\mathrm{SU}_L(2)\times\mathrm{U}_1(1)\underset{v_L^2}{\rightarrow}\mathrm{U}_{em}\\ &V^2\gg v_R^2\gg v_L^2\,.\end{aligned}$$

2. For the symmetry breaking pattern in two stages, viz
$$\mathrm{SU}_L(4)\times\mathrm{U}_X(1)\underset{v_R^2}{\rightarrow}\mathrm{SU}_L(2)\times\mathrm{U}_1(1)\underset{v_L^2}{\rightarrow}\mathrm{U}_{em}\,.$$
There is no need to introduce, 15-plet Higgs multiplet. This is an attractive possibility

3. In the decoupling limit, i.e. $g_X\to\infty$, V_μ is decoupled and the minimum Higgs required for the symmetry breaking are Higgs multiplets with $Y_X = 0$ viz Φ, ϕ_2 and ϕ_3. There are then two neutral bosons Z and Z'. It effectively goes over to $\mathrm{SU}_L(4)$.

4. The ratio between the vacuum expectation values v_R and v_L is given by
$$\frac{v_R}{v_L}\sim\frac{m_X}{m_Z}\approx 33\,,\qquad \text{for } m_X = 3\,\mathrm{TeV}$$
Using $v_L = 264\,\mathrm{GeV}$,
$$v_R = 33v_L = 8.7\,\mathrm{TeV}\,.$$

5. In eq. (2.18), the mass terms $\mathcal{K}_R$, $\mathcal{K}$, $\mathcal{K}'$ arises from the vacuum expectation values of Higgs multiplets S_{ij} and S'_{ij} (section 3), which are required to give masses to leptons and large Majorana mass to neutrinos N_l. We will assume $\mathcal{K}$, $\mathcal{K}'\ll v_L$, so that their contributions to vector bosons mass matrix can be neglected.The mixing between W_L and W_R is determined by an angle
$$\epsilon\sim\frac{\mathcal{K}\mathcal{K}'}{v_R^2}=\frac{\mathcal{K}\mathcal{K}'}{v_L^2}\frac{v_L^2}{v_R^2}$$
which is negligiblly small. In the limit of zero neutrino mass, $\mathcal{K}' = 0$, $\epsilon = 0$ i.e. left-handed and right-handed currents are completely decoupled from each other.

JHEP12(2004)013

3. Lepton sector of the group $\mathrm{SU}_L(4) \times \mathrm{U}_X(1)$

For leptons, since Y_X is zero; the gauge invariant lagrangian is similar to that given in ref. [9];

$$
\begin{aligned}
\mathcal{L}_l &= \bar{F}_\ell i\gamma^\mu \left[\partial_\mu + i\frac{g}{2}\vec{\lambda}\cdot\vec{W}_\mu\right] F_\ell \\
&\to -\frac{g}{\sqrt{2}}\left[\bar{\nu}_{\ell L}\gamma^\mu \ell_L W^-_{L\mu} + \text{h.c.} + \bar{N}'_{\ell R}\gamma^\mu \ell_R W^-_{R\mu} + \text{h.c.}\right] - \\
&\quad -\frac{g}{2}\left[\bar{\ell}_L\gamma^\mu\ell_L\left(W_{3L\mu}+\frac{1}{\sqrt{2}}B_{1\mu}\right) + \bar{\nu}_{\ell L}\gamma^\mu\nu_{\ell L}\left(-W_{3L\mu}+\frac{1}{\sqrt{2}}B_{1\mu}\right)\right] + \\
&\quad +\bar{N}'_{\ell R}\gamma^\mu N'_{\ell R}\left(W_{3R\mu}-\frac{1}{\sqrt{2}}B_{1\mu}\right) - \bar{\ell}_R\gamma^\mu\ell_R\left(W_{3R\mu}+\frac{1}{\sqrt{2}}B_{1\mu}\right) - \\
&\quad -\frac{g}{\sqrt{2}}\left[-\bar{\ell}^c_R\gamma^\mu N'_{\ell L}X^-_{1\mu} - \bar{\nu}^c_{\ell R}\gamma^\mu N_{\ell L}X^0_{2\mu} - \bar{\ell}^c_R\gamma^\mu\ell_R Y^{--}_{1\mu} + \bar{\nu}^c_{\ell R}\gamma^\mu\ell_R Y^-_{2\mu} + \text{h.c.}\right] \\
&= -\frac{g_2}{\sqrt{2}}\left[\bar{\nu}_{\ell L}\gamma^\mu \ell_L W^-_{L\mu} + \text{h.c.} + \bar{N}'_{\ell R}\gamma^\mu \ell_R W^-_{R\mu} + \text{h.c.}\right] - \\
&\quad -g_2\sin\theta_W\left(-\bar{\ell}\gamma^\mu\ell\right)A_\mu - \frac{g_2}{2\cos\theta_W}\left[\left(\bar{\nu}_{\ell L}\gamma^\mu\nu_{\ell L} - \bar{\ell}_L\gamma^\mu\ell_L\right) + 2\sin^2\theta_W\bar{\ell}\gamma^\mu\ell\right]Z_\mu - \\
&\quad -\frac{1}{2}g_1\Bigg[\left(\bar{N}_{\ell R}\gamma^\mu N_{\ell R} - \bar{\ell}_R\gamma^\mu\ell_R\right) + \\
&\qquad\qquad +\left(1+\frac{1}{2}\frac{g_2^2}{g_X^2}\right)\tan^2\theta_W\left(\bar{\nu}_{\ell L}\gamma^\mu\nu_{\ell L} - \bar{\ell}_L\gamma^\mu\ell_L + 2\bar{\ell}\gamma^\mu\ell\right)\Bigg]Z'_\mu - \\
&\quad -\frac{1}{2}g_1\left[-\frac{1}{2}\frac{g_2^2}{g_X^2}\tan^2\theta_W\left(2\bar{\ell}\gamma^\mu\ell + \bar{\nu}_{\ell L}\gamma^\mu\nu_{\ell L} - \bar{\ell}_L\gamma^\mu\ell_L\right)\right]Z''_\mu - \\
&\quad -\frac{g_2}{\sqrt{2}}\left[-\bar{\ell}^c_R\gamma^\mu N'_{\ell R}X^-_{1\mu} - \bar{\nu}^c_{\ell R}\gamma^\mu N'_{\ell R}X^0_{2\mu} - \bar{\ell}^c_R\gamma^\mu\ell_R Y^{--}_{1\mu} + \bar{\nu}^c_{\ell R}\gamma^\mu\ell_R Y^-_{2\mu} + \text{h.c.}\right].
\end{aligned}
\tag{3.1}
$$

In the limit $g_X \to \infty$, it goes over to $\mathrm{SU}_L(4)$.

Note that N'_ℓ is not a mass eigenstate. It is related to mass eigenstate N_ℓ as follows

$$
\begin{gathered}
N'_\ell = U_{\ell\ell'}N_\ell \\
\begin{pmatrix} N'_e \\ N'_\mu \\ N'_\tau \end{pmatrix} = U \begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix}, \qquad UU^T = 1\,.
\end{gathered}
\tag{3.2}
$$

To give masses to leptons, and large Majorana mass to the neutrino N_ℓ, we introduce two Higgs multiplets S_{ij} and S'_{ij} belonging to a symmetric representation 10 of $\mathrm{SU}_L(4)$. This is done by giving the expectation values as follows:

$$
\langle S_{33}\rangle = \mathcal{K}_R\,, \qquad \langle S'\rangle = \begin{pmatrix} 0 & 0 & 0 & -\mathcal{K} \\ 0 & 0 & \mathcal{K}' & 0 \\ 0 & \mathcal{K}' & 0 & 0 \\ -\mathcal{K} & 0 & 0 & 0 \end{pmatrix}. \tag{3.3}
$$

The scalars S_{ij} and S'_{ij} have $Y_X = 0$; thus they will be coupled only to leptons and not to quarks. We shall assume that Yukawa couplings of S_{ij} and S'_{ij} are not very weak but have

JHEP12(2004)013

normal strengths. Thus $\mathcal{K}$, $\mathcal{K}'$, $\mathcal{K}_R$ are very small compared to the vacuum expectation values of ϕ's viz $\mathcal{K}$, $\mathcal{K}' \ll v_L$; $\mathcal{K}_R \ll v_R$. In this case, we may neglect their contributions to masses of vector bosons.

The Yukawa couplings of these scalars to leptons are given by

$$-f_\ell F_{\ell i}^T C^{-1} F_{\ell j} S_{ij} - f'_\ell F_{\ell i}^T C^{-1} F_{\ell j} S'_{ij} + \text{h.c.} \tag{3.4}$$

The lepton mass term is then given by

$$\mathfrak{m}_\ell = \left[f'_\ell \mathcal{K} \bar{\ell}_L \ell_R + f'_\ell \mathcal{K}' \bar{\nu}_{\ell L} N'_{\ell R} - f_\ell \mathcal{K}_R N'^T_{\ell R} C^{-1} N'_{\ell R} + \text{h.c.} \right]. \tag{3.5}$$

From eq. (3.5), the neutrino mass term can be written in the form:

$$\begin{aligned} \mathfrak{m}_\nu = \mathcal{K}' \Big\{ & f'_e \bar{\nu}_{eL} \left(U_{ee} N_{eR} + U_{e\mu} N_{\mu R} + U_{e\tau} N_{\tau R} \right) + \\ & + f'_\mu \bar{\nu}_{\mu L} \left(U_{\mu e} N_{eR} + U_{\mu\mu} N_{\mu R} + U_{\mu\tau} N_{\tau R} \right) + \\ & + f'_\tau \bar{\nu}_{\tau L} \left(U_{\tau e} N_{eR} + U_{\tau\mu} N_{\tau R} + U_{\tau\tau} N_{\tau R} \right) \Big\} + \\ & + \mathcal{K}_R \left[f_e \bar{N}^c_{eL} N_{eR} + f_\mu \bar{N}^c_{\mu L} N_{\mu R} + f_\tau \bar{N}^c_{\tau L} N_{\tau R} \right] + \text{h.c.} \end{aligned} \tag{3.6}$$

Note that we have obtained eqs. (3.5) and (3.6) from eq. (3.4) by using the relation $\psi_R^T = -\bar{\psi}_L^c C$.

It is clear from eq. (3.6) that the see-saw mechanism is a natural consequence of this model. The flavor mixing for neutrinos is built in feature of this model. The CKM type matrix U can accomodate many features of the neutrino oscillations which has been of considerable interest recently. The lepton number violation occurs at a TeV scale. These processes are mediated by vector bosons X, Y.

Some interesting lepton number violating processes that can occur at tree level, through the exchange of Y_2 and Y_1 bosons are:

$$\begin{aligned} \mu^- &\to \bar{\nu}_\mu + e^- + \nu_e \qquad \Delta L_\mu = -2, \Delta L_e = 2 \\ \mu^+ e^- &\to \mu^- e^+ \\ \bar{\nu}_\mu e^- &\to \bar{\nu}_e \mu^- \end{aligned}$$

The experimental limit on the decay $\mu^- \to \bar{\nu}_\mu + e^- + \nu_e$

$$R = \frac{\Gamma\left(\mu^- \to \bar{\nu}_\mu + e^- + \nu_e\right)}{\Gamma\left(\mu^- \to all\right)} < 1.2 \times 10^{-2};$$

Since

$$R \propto \left(\frac{m_W}{m_Y}\right)^4,$$

the above limit implies

$$m_Y > 3 m_W .$$

This implies a lower limit for the unification scale viz $m_X > 3m_W$. Thus the upper limit for the unification scale (5.8 TeV) is much above the present experimental limit. Before we close this section, the following comments are in order.

The mass scale $\mathcal{K}$, $\mathcal{K}'$ and $\mathcal{K}_R$ are essentially determined by masses of charged leptons, light neutrinos and heavy Majorana neutrinos. The see-saw mechanism gives the light neutrino mass:

$$\begin{aligned} m_{\nu_l} &\sim \frac{(f_l'\mathcal{K}')^2}{f_l\mathcal{K}_R} = \frac{m_l^2}{f_l\mathcal{K}_R}\left(\frac{\mathcal{K}'}{\mathcal{K}}\right)^2 \\ f_l\left(\frac{\mathcal{K}}{\mathcal{K}'}\right)^2 &\sim \frac{m_l^2}{m_{\nu_l}\mathcal{K}_R} \end{aligned} \tag{3.7}$$

The atmospheric neutrino anomly implies $\Delta m_\nu^2 \sim 3\times 10^{-3}\,\mathrm{eV}^2$ and sets the scale of possible neutrino masses at $\geq 5\times 10^{-2}\,\mathrm{eV}$. On the other hand WMAP data [13] give $m_\nu < 0.23\,\mathrm{eV}$. Hence for $m_e = 0.51\,\mathrm{MeV}$, we get

$$1 < f_e\left(\frac{\mathcal{K}'}{\mathcal{K}}\right)^2 < 5 \qquad \text{for } \mathcal{K}_R = 1\,\mathrm{TeV}\,. \tag{3.8}$$

Hence there is no problem of heirarchy between the scales $\mathcal{K}$ and $\mathcal{K}'$

4. Quark sector of the group $\mathrm{SU}_L(4)\times \mathrm{U}(1)$

For the first generation (for example), the interaction lagrangian for quarks is given by

$$\begin{aligned} \mathcal{L}_q = &-\frac{g_2}{\sqrt{2}}\left[\bar{u}_L\gamma^\mu d_L' W_{L\mu}^- + \bar{D}_L\gamma^\mu H_{dL}W_{L\mu}^- + \text{h.c.}\right] - g_2\sin\theta_W J_{em}^\mu A_\mu - \\ &-\frac{g_2}{2\cos\theta_W}\left[\bar{u}_L\gamma^\mu u_L - \bar{d}_L\gamma^\mu d_L - 2\sin^2\theta_W J_{em}^\mu\right] Z_\mu - \\ &-\frac{1}{2}g_1\left[\left(\bar{D}_L\gamma^\mu D_L - \bar{H}_{dL}\gamma^\mu H_{dL}\right) + \left(1+\frac{1}{2}\frac{g_2^2}{g_X^2}\right)\tan^2\theta_W\left(\bar{u}_L\gamma^\mu u_L - \bar{d}_L\gamma^\mu d_L - 2J_{em}^\mu\right)\right] Z_\mu' - \\ &-\frac{g_1}{2}\left[\frac{1}{2}\frac{g_2^2}{g_X^2}\tan^2\theta_W\left(-\bar{d}_L\gamma^\mu d_L + \bar{u}_L\gamma^\mu u_L - 2J_{em}^\mu\right)\right] Z_\mu'' \\ &-\frac{g_2}{\sqrt{2}}\left[\bar{u}_L\gamma^\mu D_L X_{1\mu}^- + \bar{d}_L'\gamma^\mu D_L X_{2\mu}^0 + \bar{u}_L\gamma^\mu H_{dL}Y_{1\mu}^{--} + \bar{d}'\gamma^\mu H_{dL}Y_{2\mu}^{--} + \text{h.c.}\right] \end{aligned} \tag{4.1}$$

where

$$J_{em}^\mu = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{D}\gamma^\mu D - \frac{4}{3}\bar{H}_d\gamma^\mu H_d\,. \tag{4.2}$$

The quarks acquire masses from their Higgs

$$\phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_1^- \\ \phi_1'^- \\ \phi_1^{--} \end{pmatrix} \qquad \langle\phi_1^0\rangle = v_1 = v_L\sin\alpha$$

$$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \\ \phi_2'^0 \\ \phi_2^- \end{pmatrix} \qquad \langle\phi_2^0\rangle = v_2 = v_L\cos\alpha$$

JHEP12(2004)013

$$\phi_3 = \begin{pmatrix} \phi_3^+ \\ \phi_3^0 \\ \phi_3'^0 \\ \phi_3^- \end{pmatrix} \qquad \langle \phi_3'^0 \rangle = v_3 = v_R \cos\beta$$

$$\phi_4 = \begin{pmatrix} \phi_4^{++} \\ \phi_4^+ \\ \phi_4'^+ \\ \phi_4^0 \end{pmatrix} \qquad \langle \phi_4^0 \rangle = v_4 = v_R \sin\beta\,. \tag{4.3}$$

Let us write down the Yukawa couplings of scalars ϕ's with the quarks: (here we depict the couplings for the first generation)

$$\mathcal{L}_Y = f_u \bar{F}_d \phi_1 u_R + f_d \bar{F}_d \phi_2 d'_R + f_d \bar{F}_d \phi_2 D_R + f_D \bar{F}_d \phi_3 D_R + f_D \bar{F}_d \phi_3 d'_R + f_{H_d} \bar{F}_d \phi_4 H_{d_R} + \text{h.c.} \tag{4.4}$$

Note that while the scalars ϕ_1 and ϕ_4 are coupled to u_R and H_{d_R} respectively; the scalars ϕ_2 and ϕ_3 are coupled to both d_R and D_R, giving off-diagonal mass terms for d_R and D_R. In order to avoid this, we require the Yukawa couplings to be invariant under a discrete symmetry: under which

$$\begin{aligned} \phi_1, \phi_2 &\to \phi_1, \phi_2, \\ \phi_3, \phi_4 &\to -\phi_3, -\phi_4, \\ F_d &\to F_d, u_R, d_R \to u_R, d_R \\ D_R, H_{d_R} &\to -D_R, -H_{d_R}\,. \end{aligned}$$

In this way, the quarks acquire masses given below

$$\begin{aligned} m_u &= f_u v_L \sin\alpha, m_d = f_d v_L \cos\alpha \\ m_D &= f_D v_R \cos\beta, m_{H_d} = f_{H_d} v_R \sin\beta\,. \end{aligned} \tag{4.5}$$

Thus, the quarks D and H_d are superheavy.

Now we have two types of Higgs coupled to down quarks. Thus we must address the question of suppression of flavor changing neutral currents (FCNC). At tree level, there are no FCNC. However, they may appear at loop level. FCNC can be eliminated by replacing D and S by D' and S', where D' and S' are related to mass eigenstates D and S with same Cabibbo rotation as for d' and s'. This is a reasonable assumption as D and S are replicas of d and s at a higher mass scale.

To conclude, The electroweak unification model based on the group $\mathrm{SU}_L(4) \times \mathrm{U}_X(1)$ has following features: In the leptonic sector, it contains heavy right-handed neutrinos N_e, N_μ and N_τ. The see-saw mechanism for the neutrino mass matrix is built in the model. $\Delta L_\mu = \pm 2$, $\Delta L_e = \mp 2$ processes can occurs due to exchange of heavy bosons X and Y. In the quark sector we have six extra heavy quarks; three of them viz (U, D, S) have charges $(2/3, -1/3, -1/3)$ and are mirrors of (u, d, s) quarks. However, these quarks would decay to light quarks by β-decay with the emission of right-handed Majorana neutrino at tree level through the exchange of X-bosons. A typical process is depicted below $(\Delta L_e = 2)$

$$D \to u + e^- + N_e\,.$$

The superheavy quarks (U, D, S) can form bound states with the light quarks u, d, s, c and b. Thus this model predict a replica of existing hadrons at a scale of few hundred GeV to TeV. The superheavy quarks H_d, H_s, and T, have exotic charges $(-4/3, -4/3, 5/3)$. If these quarks have masses greater than Y bosons, they would decay quickly to light quarks by a process of the form $H_d \to u + Y_1^{--}$. Even if they have masses below Y-bosons, they are expected to decay quickly to light leptons by a typical process: (neutrinoless β-decay; $\Delta L_e = 2$)

$$H_d \to u + l^- + l^- \,.$$

Thus H_d, H_s, and T may not live long enough to form bound states with light quarks.

Finally, from the interaction lagrangians given in eqs. (3.1) and (4.1), it is clear that for $g_2^2/g_X^2 \ll 1$, we get effectively interaction lagrangians of group $\mathrm{SU}_L(4)$. Thus for $g_2^2/g_X^2 \ll 1$, it is still possible to have unification at TeV scale.

In the end, it is interesting to note that any viable extension of the standard model $(\mathrm{SU}(3)_c \times \mathrm{SU}_L(2) \times \mathrm{U}_Y(1))$ is not possible without extending the existing elementary consituents of matter (viz six leptons and six quarks). In this sense, the standard model is unique if there are no additional constituents of matter.

Note added: after putting the paper on the `arXiv`, we were informed by Dr. R. Foot that the group $\mathrm{SU}_L(4)$ originally considered by us [9] has been extended earlier to $\mathrm{SU}_L(4) \times \mathrm{U}(1)$ [11, 12] to include fractionally charged quarks. These extensions do not contain any reference to our earlier work [9]. The motivation of our work is some what different, although there is overlap with ref. [11, 12]. Moreover, in our version, see-saw mechanism for neutrino mass matrix is a natural consequence.

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